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Economic Scenario Generators

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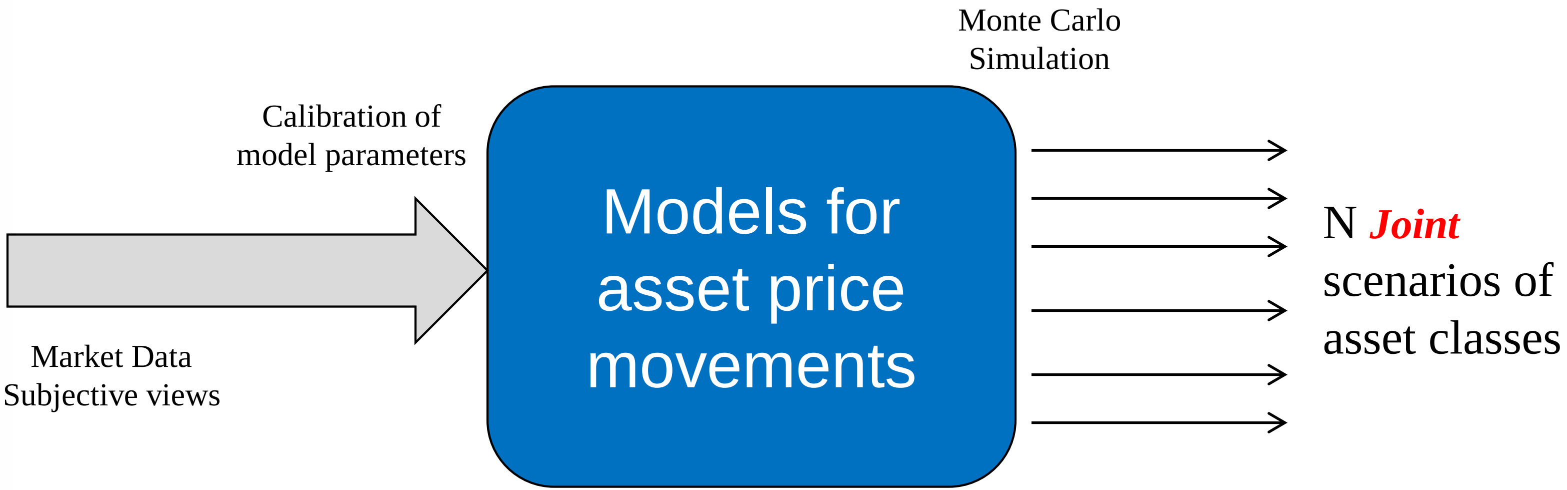
Agenda

- What is an ESG
- Types of ESG
- Why do we need an ESG
- Risk neutral ESGs
  - Models
  - Calibration
  - Validation
- Challenges in Indian markets

Introduction to Economic Scenario Generator

- Future is unknown
- We may have expectations about the future but we are never certain about it
- An ESG is a tool which
  - Uses Monte Carlo simulation to
  - Generate numerous simulations of economic variables
  - Over multiple time periods
- Average of the simulations converge to our expectation

Introduction to Economic Scenario Generator



Types of ESG

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Risk Neutral (RN)	Real World (RW)
• **<u>Market consistent</u>**: Parameters of underlying models are calibrated such that ***economic scenarios*** are ***consistent with market prices***	• **<u>Subjective</u>**: Economic scenarios modeled to reflect subjective views about the future evolution of the markets
• **<u>Risk neutral:</u>** Scenarios are modeled ensuring that no arbitrage allowed. All financial instruments will have the same ***expected return*** which is ***equal to the risk free rate***	• **<u>Not market consistent:</u>** Economic scenarios are not consistent with current market prices
• Individual scenarios results do not hold any significance	• Incorporate risk premia in asset returns
• Used for pricing and valuation only	• Individual scenarios can be used for analysis
• Not intended to reflect real world expectations	• Used for activities which require realistic forward looking projections

Why do we need an ESG?

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Market consistent valuation of options & guarantees RN

Formulation of ALM/ investment strategies RW

Risk management/ economic capital calculation RW

Business planning/ Capital planning RW

Pricing RN

Why do we need an ESG?

As per APS 10 , Embedded Value should

- Allow for time value of Financial Options & Guarantees
- Allowance should be based on stochastic techniques
- Economic assumptions should be in line with capital market prices of similar traded cash flows ⟺ Market consistent
- As identical traded options may not exist, we need a Market Consistent/ Risk Neutral ESG

Types of options & guarantees embedded in life insurance products

Non-linear payoffs/ guarantees need to valued using an ESG

Examples

- **Minimum return guarantee** in participating/Unit linked products:
  - Guarantees in par products are non-linear
  - Upside shared between SH and PH
  - Downside fully borne by SH

- **Surrender option**
  - Similar to an American option
  - Can be exercised at any point of time during the contract depending on the perceived value of the option

Types of options & guarantees embedded in life insurance products

Examples

- **Paid-Up option**
  - Similar to Bermudan options
  - Can be exercised at premium payment dates

- Valuation of options is tricky as it requires assumptions about “Option exercise strategy/ policy holder behavior”

Risk neutral ESGs

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Selection of asset models

Calibration of model parameters

Generation of economic scenarios using Monte Carlo simulation techniques

Validation

Asset Models

Very generically, all asset models are of the form:

- $dS = a(t,S_t)*dt + \sigma(t,S_t)*dW_t$

Where a & σ are the drift and diffusion functions and

W_t is a Weiner process

- W_t has Gaussian increments, i.e. the distribution of $W_t - W_s \sim N(0,t\text{-}s)$
- The increments are independent of each other
- $W_0 = 0$

Asset Models: Interest rate models

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Interest rate models

- **SHORT RATE MODELS**: Model the behavior of instantaneous spot/ forward rates
- **MARKET MODELS**: Model the behavior of forward rates observed in the market

Short rate models

- **ONE FACTOR**:

  Example – Hull White 1-F model

  $dr_t = [\theta(t) - ar_t]*dt + \sigma*dW_t$,

  where a and σ are positive constants and θ(t) is chosen so that the model exactly matches the term structure of interest rates

- **TWO FACTOR/ MULTI FACTOR**

Asset Models: Interest rate models

Short rate models

TWO FACTOR:

Example – Hull White 2F model/ G2++ model

$dr_t = (\theta(t) + u(t) - ar_t)*dt + \sigma_1*dW_{1,t}$

$du(t) = b*u_t*dt + \sigma 2*dW_{2,t}$

$dW_{1,t}*dW_{2,t} = \rho*dt$

where a, b, σ_1, σ_2 and ρ are positive constants and $\theta(t)$ is chosen so that the model exactly matches the term structure of interest rates

1-F versus 2-F models

- Easier analytical tractability in 1-F models
- However, the resultant spot rates for all maturities are perfectly correlated with each other. Thus a **one factor model allows only for parallel shifts of the yield curve** and no shape changes are possible.

Asset Models: Interest rate models

Market Models: Libor Market Model

Libor Market Model:

Most widely used markets in developed markets

LMM models forward rates which are observable in the market

Each forward rate F(t, T) follows a process where the drift is dependent on the other forward rates

Correlation between different forward rates is also allowed for.

Leads to a better fitting of volatility structure of interest rates

Market models versus Short rate models

- Market models are relatively difficult to implement
- Market models need a lot many data points for calibration
- Allow for a better fitting of volatility structure of interest rates

Asset Models: Equity

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Black Scholes Merton model

$ds_t = \mu * S_t * dt + \sigma * S_t * dW_t$

$S_{t+1} = S_t * \exp[(\mu - \sigma^2/2) * t + \sigma * W_t]$

Lognormal is the simplest model for Equity prices.
It assumes a constant volatility structure
Unable to replicate market prices of out of the money options

There is a trade off between the complexity of model & the goodness of fit. Models need to be chosen based on the requirements of the task in hand

Risk neutral ESG Calibration

Calibration is the process by which the parameters of the chosen models are estimated.

Objective Calibration criteria: Model fits the observed market prices of options
Options used: equity calls/puts, interest rate caps/ floors, swaptions

Model parameters are usually calibrated using
- Analytical expressions for option prices (for simplistic models)
- Numerical methods – Building Trinomial trees (for most models)

- Illustration for building trinomial trees has been given in the paper and is also available online

- Codes\ Packages for calibration exist in open source softwares like Python & R

Generation of simulations & Validation

Generation of simulations:

- Monte carlo simulation techniques applied on the calibrated models
- For simulating joint behavior of economic variables
  - Correlation between asset classes is estimated based on historical data
  - Cholesky decomposition is used to generate correlated random numbers

Validation:

1. Risk neutrality – Martingale test: Average of discounted value of any asset over the simulated paths should be equal to current market price of the asset

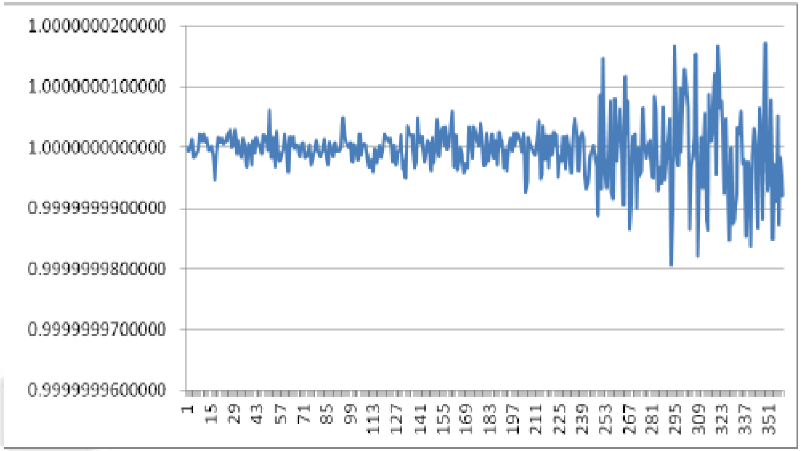
$$\sum_{i=1}^{N=\text{Simulated ESG paths}} \frac{D_{i,t} * \left(1 + R_{t,i}^{cum}\right)}{N} = 1$$

Validation

Cumulative Returns on NIFTY						
SCENARIO\ TIME	0	1	2	3	4	5
1		103%	122%	142%	115%	81%
2		89%	88%	125%	134%	146%
3		132%	121%	153%	166%	193%
4		110%	113%	131%	135%	165%
5		112%	118%	104%	115%	112%
6		100%	104%	108%	128%	134%
7		106%	145%	217%	235%	197%
8		105%	111%	99%	119%	187%
9		125%	148%	161%	185%	206%
10		98%	100%	106%	89%	100%

Path wise Discount Factors						
SCENARIO\ TIME	0	1	2	3	4	5
1	1	95%	87%	84%	81%	79%
2	1	91%	80%	70%	62%	54%
3	1	89%	79%	74%	71%	70%
4	1	92%	88%	78%	72%	66%
5	1	92%	85%	76%	73%	72%
6	1	91%	75%	65%	59%	55%
7	1	90%	78%	68%	61%	55%
8	1	92%	86%	77%	72%	67%
9	1	94%	91%	82%	69%	58%
10	1	94%	86%	78%	72%	65%

Martingale Test Statistic						
SCENARIO\ TIME	0	1	2	3	4	5
1		98%	107%	119%	93%	64%
2		81%	70%	88%	83%	80%
3		118%	96%	113%	118%	135%
4		101%	99%	102%	97%	109%
5		103%	101%	79%	84%	81%
6		92%	78%	70%	75%	74%
7		96%	114%	149%	143%	108%
8		97%	96%	77%	85%	126%
9		118%	135%	132%	127%	120%
10		92%	86%	83%	64%	65%
Average*		1.00	1.00	1.00	1.00	1.00



Validation

Market consistency/Goodness of fit test:

- Comparison of prices of traded instruments
  - computed using ESG simulation output
  - actual traded prices

$$P(t=0) = \frac{\sum_{i=1}^{N=\text{all paths in ESG output}} D_i X_i}{N}$$

- LHS is the actual price of an option
- RHS is the price computed using ESG output (Average of the discounted value of option payoff)

Challenges in Indian markets

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- Data required for risk neutral ESG calibration
  - Yield curve
  - Equity Implied option volatility – NIFTY options
  - Implied volatility on interest rate options – Swaptions, Interest Rate caps & floors, bond options

- Challenges
  - Equity implied option volatility:
    - Only short tenure options are available
    - Implied volatilities of options varies by tenure of the option

  - Interest rate implied option volatilities:
    - Interest rate options are traded only OTC
    - Data is thin and difficult to obtain

Challenges in Indian markets

- Possible solutions - Equity
  - **Assume a constant volatility**: leads to an over-estimation of short dated options and under-estimation of long date options

  - **Functional form for implied volatility:**
    - Use observed implied volatility data
    - Use a long term volatility assumption (Based on realized long term volatility)
    - Impose a functional form for the volatility term structure
    - Interpolate/ Extrapolate volatility for tenures to be used for calibration

Challenges in Indian markets

- Possible solutions – Interest Rate volatilities

  - Bond options are already trading on exchanges – short term only

  - Use of data from other developed markets:
    - Use the implied volatility surface from developed markets
    - Compute the relative value factors of implied versus realized volatility
    - Apply these factors to realized volatility of forward/ swap rates

  - Directly use the forward/ swap volatilities observed in Indian markets and fit volatility estimation models like GARCH to estimate forward looking volatilities